Appendices

APPENDIX 1 – MINIMUM-SWR RESISTANCE EQUATION

I have never seen a reference to the minimum-SWR resistance in relation to terminating loads in any of the engineering or academic literature. Therefore, I believe I am the originator of the relationships represented by Eqs 5-1 and 5-2, discussed in Sec 11.3, and the derivation given here.

Theorem:

When a transmission line of characteristic impedance Z_C is terminated with an impedance Z = R + *j*X, for every value of X there is corresponding value of R which yields the minimum value of SWR. For minimum SWR when R = Z_C, X must be zero.

Prove that $r = \sqrt{x^2 + 1}$ (see Fis Apendix 1-1)

where

r = normalized resistance component of Z yielding minimum SWR

x = normalized reactance component of Z

r' = radius of resistance circle = $\frac{1}{r+1}$

U_C = center of resistance circle = $\frac{r}{r+1}$

$\frac{1}{x}$ = center of reactance circle

ρ = magnitude of reflection coefficient

$\tan\theta = \frac{1}{1/x} = x = \frac{\rho}{r'}$

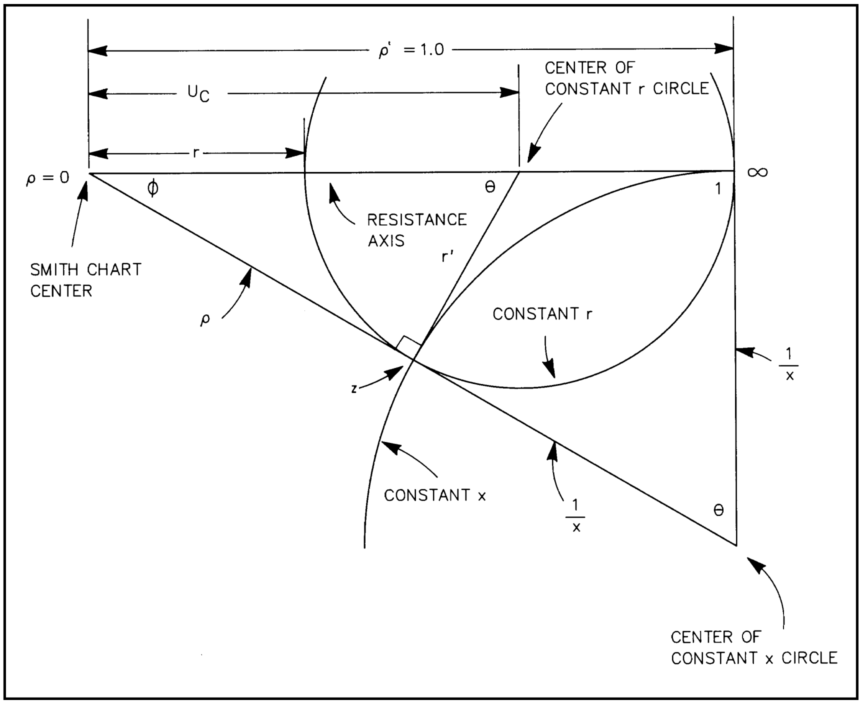

Fig Appendix 1-1 – Smith Chart geometry for Appendix 1 proof of minimum-SWR equation.

$$\rho + \frac{1}{x} = \sqrt{(1/x)^2 + 1^2} = \frac{\sqrt{x^2+1}}{x}$$

$$\rho = \frac{\sqrt{x^2+1} - 1}{x}$$

$$r' = \frac{\rho}{x} = \frac{\sqrt{x^2+1} - 1}{x^2} = \frac{1}{r+1}$$

Inverting,

$$\frac{1}{r'} = \frac{x^2}{\sqrt{x^2+1} - 1} = r + 1$$

Clearing radical in denominator,

$$\frac{x^2}{\sqrt{x^2+1} - 1} \times \frac{\sqrt{x^2+1} + 1}{\sqrt{x^2+1} + 1} = r + 1$$

Multiplying and canceling,

$$\frac{x^2\sqrt{x^2+1}+x^2}{x^2+1-1}=r+1$$

$$\sqrt{x^2+1}+1=r+1$$

$$\sqrt{x^2+1}=r$$

Therefore the theorem is proved.