APPENDIX 9A – REVEALING INHERENT ERRORS IN MEASUREMENTS OF NETWORK OUTPUT IMPEDANCE WHEN LOOKING REARWARD INTO THE NETWORK

Sec APA.1 Background

As in previous chapters that clarified misconceptions concerning SWR, reflections, and conjugate matching, the goal of this Appendix is to clarify misconceptions concerning the procedure for calculating and measuring the output impedance of networks and transmission lines when the input impedance is known. Misuse of procedure in these calculations and measurements has given rise to a prevalent misconception that a conjugate match cannot exist in a system comprising networks and transmission lines having attenuation. If this were true a conjugate match could never exist in practical systems, because all real networks and transmission lines have attenuation. The misconception arises from improper application of Axiom 4 while attempting to determine the output impedance of networks and transmission lines when connected to real sources in contrast to that of the classical generator. The reason is that there is a potential for conflict between Axioms 1 and 2 with Axiom 4 that arises when comparing a system with a non-dissipative source with one having the dissipative source of the classical generator. The potential for conflict is especially true when attempting to determine the output impedance of a network by measuring rearward from the output terminals. A clue to the conflict is that the output impedance of a lossy network or transmission line obtained by measuring rearward from the output with the input terminated in a dissipative resistor is not the same as the actual output impedance of the network when the network is in normal operation. Thus the impedance obtained by rearward measurement is incorrect. In addition, the same incorrect value of output impedance is obtained by using the attenuation factor incorrectly in *calculating* the output impedance when the input impedance is known. The difference between the correct output impedance and that obtained by rearward measurement (looking backward), and thus the error, is proportional to the amount of attenuation; the error is zero when attenuation is zero.

Sec APA.2 Explanation

We'll now explain why the incorrect application of Axiom 4 and incorrect use of the attenuation factor leads to the misconception that a conjugate match cannot exist when network components and transmission lines have attenuation. It must be first understood that according to Axioms 1 and 2 there is a conjugate match when all available power from the source is being delivered to the load. During this condition the output impedance of the network or line is the conjugate of the load impedance by definition. However, while keeping in mind that the source resistance in the classical generator is inherently dissipative, the incorrect application of Axiom 4 arises when attempting to determine the output impedance of a network or line by measuring rearward from the output. During this measurement the input of the network or line is terminated in a real dissipative resistor that replaced the non-dissipative source resistance of the same value. The reason this procedure is incorrect is that the output impedance of a lossy network or transmission line with attenuation while delivering power to the load is not the same as the impedance obtained by measuring rearward from the output terminals with the input terminated in a dissipative resistor. Consequently, the impedance obtained by rearward measurement is not the conjugate of the load impedance. Unfortunately, because the impedance obtained by rearward measurement of a lossy network is not the conjugate of the load impedance it is often assumed incorrectly that there is no conjugate match at the junction of the network output and its load. It is indeed disastrous that this incorrect measurement procedure to determine the output impedance has resulted in the misconception that no conjugate match exists, when in fact it does exist. So let us examine the reason why this incorrect procedure produces the incorrect value of output impedance.

The principal concerns in obtaining the correct network output impedance when attenuation is present are the direction of travel through the network (or along a transmission line) and the correct use of the attenuation factor during the measurement. If the network or line is lossless, the direction of travel is irrelevant, because the resulting impedances are identical in traveling in either direction. However, when there is attenuation in the network or line, the direction of travel and correct use of the attenuation factor (which depends on the direction of travel) are of paramount importance in obtaining the correct output impedance. The basic reason direction of travel and correct attenuation factor are important is that in returning from a mismatched load terminating the network or line, the voltage and current reflected at the load mismatch decrease while traveling toward the source due to the attenuation. Thus the magnitude of the voltage and current reflection coefficients, and thus the SWR, is less at the input than at the output where the mismatched load terminates the network or line. Consequently, the impedance appearing at the input of the line is different than if the line were lossless. These differences of SWR and impedance due to attenuation are the basis for the problem concerning the direction of travel along the line, and the correct application of the attenuation factor during measurements or calculations to obtain the correct values of input and/or output impedance. As we will see, measuring rearward from the network output (source power now applied at the output) the attenuation factor applies in the opposite direction to that when the source power is applied at the input.

Sec APA.3 Definition and Use of Attenuation Factor

We'll now define 'attenuation factor', and explain how it's used correctly in measurements and calculations. The attenuation factor applied here is the decimal equivalent of the two-way voltage (and current) attenuation in dB. Its inherent effect is to change the magnitude of the voltage (and current) reflection coefficients at either end of a line or network in proportion to the amount of its attenuation. Whether the magnitude is increased or decreased at a given end depends on whether the calculation begins at the output or input.

If the magnitude of the reflection coefficient at the output is known, we start the calculation at the output to determine the reflection magnitude at the input using the expression $\rho_{IN} = \rho_{OUT}\, e^{-2\alpha}$, yielding a decrease in magnitude at the input. Conversely, when the reflection magnitude at the input is known, we start the calculation at the input to determine the magnitude at the output, using the expression $\rho_{OUT} = \rho_{IN}\, e^{2\alpha}$, yielding an increase in magnitude at the output. Note that the change in sign of the exponent in the expressions accounts for the difference in direction of travel along the network or line, with the negative exponent applying to the direction from output to input, and the positive exponent applies while going from input to load. The legends for the terms in the expressions are:

ρ_{IN} = magnitude of the reflection coefficients at the input,
ρ_{OUT} = magnitude of the reflection coefficients at the output,
e = 2.71828, the base of natural logarithms, and
α = attenuation in nepers = dB/20 $\log_e$ = dB/8.6859
(1 neper = 1/8.6859 dB = 0.115129 dB, and 1 dB = 8.6859 nepers)

However, the more simple and routine method for determining the attenuation factor is to convert the attenuation in dB to its decimal equivalent for two-way voltage attenuation (or one-way power attenuation) using the exponential expression $10^{\,dB/10}$ = the decimal attenuation factor. For example, the two-way attenuation factor for voltage and current for 0.2 dB is 0.95499, for 0.4 dB it is 0.91201, and for 1.0 dB it is 0.79433, etc. To put these values in perspective, if 100 watts were applied to a line or network having 0.2 dB of attenuation, the power available at the output would be 95.499 watts; for 0.4 dB the power available would be 91.201 watts, and for 1.0 dB, 79.433 watts.

Sec APA.4 Importance of Direction of Wave Travel

To determine SWR and terminal impedance at either line input or output, we will first be either multiplying or dividing the magnitude of the voltage reflection coefficient by the attenuation factor, depending on the starting point and direction of travel along the line. Both SWR and terminal impedance are then calculated from the modified magnitude of the reflection coefficient. (See Program HP2, "Transmission-Line Impedance Transformation," Chapter 15.) Because the magnitude of both reflection coefficient and SWR at the input are *less* than at the output due to attenuation, the known reflection coefficient at the output is **multiplied** by the decimal attenuation factor to determine the reflection coefficient at the input. Conversely, because both reflection coefficient and SWR at the output are *greater* than at the input, the known reflection coefficient at the input is **divided** by the decimal attenuation factor to determine the reflection coefficient at the output. (Alternatively, the coefficient may be multiplied by the *reciprocal* of the decimal attenuation factor.) Consequently, if we begin with the output impedance to determine the input impedance, and then return along the line to transform from the input back to the output, the output impedance determined by calculation must be identical to the output impedance appearing at the beginning. Thus the net attenuation for the round trip is 0.0 dB, and the net attenuation *factor* is 1.0. Make careful note of this compensatory relationship as the attenuation factor functions as a loss going from load to input, but the loss is compensated on returning from the load to the input. Knowledge of this relationship is crucial to understanding why measuring the impedance rearward from the network output yields the incorrect value of output impedance.

Because the direction of travel during impedance-transfer calculations and measurements determine whether the attenuation factor will be used as a multiplier or divider, we'll use logical terms to indicate which argument will be used. When the direction of travel is from load to input (calling for multiplication) we'll consider the attenuation factor **positive,** because the attenuation is **additive** going in this direction to account for the loss that causes reduction in reflection coefficient and SWR at the input. Conversely, when the direction of travel is from input to load (calling for division) we'll consider the factor **negative,** because the attenuation is **subtractive** while going in this direction to account for the increase in the reflection coefficient and SWR at the load. However, calling the attenuation factor 'negative' during this procedure does *not* imply gain. As stated earlier, correct use of *this positive/negative relationship of the attenuation factor is the crux of the problem in explaining why measurement of output impedance of the network or line by looking rearward from the output terminals yields incorrect results.*

Sec APA.5 Example Demonstrating Importance of Direction of Travel

Let's now use an example to demonstrate how the difference in direction of travel affects the results. Although the procedure applies equally well to both networks and transmission lines, we will use a transmission line in the demonstration, because the explanation will be simpler. We'll begin using a $\lambda/2$ 50-ohm transmission line having zero attenuation terminated with a pure resistance, $100 + j0$ ohms. As we know, with zero attenuation, the input impedance of the $\lambda/2$ line is a repeat of the load impedance, $100 + j0$ ohms. Thus everywhere along the line the magnitude of the reflection coefficient is 0.33333 and thus the SWR is 2:1. (See Eq 3-2) Consequently, when the line is lossless the direction of travel is irrelevant in determining the impedance at either end.

Now let's introduce a line attenuation of 1.0 dB, for which the decimal attenuation factor is 0.79433. While the impedance and SWR at the load remain at $100 + j0$ ohms and 2:1, respectively, to determine the reflection coefficient at the input the reflection coefficient at the load is multiplied by the attenuation factor 0.79433. Thus the reflection coefficient at the line input is reduced to 0.26478 by **multiplying** 0.33333 $\times$ 0.79433. This lower reflection coefficient causes a reduction of the input SWR from 2:1 to 1.720:1, and a reduction of the input impedance from $100 + j0$ to $86.0 + j0$ ohms. Thus, in traveling from the load to the input we see the effect of the **positive** line attenuation factor has reduced both the input impedance and input SWR relative to those at the mismatched load. Conversely, to determine the output impedance when the input impedance is known, it is necessary to **reverse** this procedure by **dividing** the reflection coefficient at the input by the attenuation factor (thus **negative**), also 0.79433. Thus $0.26478 \div 0.79433 = 0.33333$, the reflection coefficient at the output, the SWR of 2:1, and the original $100 + j0$-ohm load impedance. Consequently, we have shown the net attenuation during the round trip—load-to-input and returning input-to-load is zero.

Continuing with the example above, let's assume the $\lambda/2$ transmission line is matching the load impedance of 100 + *j*0 ohms to a non-dissipative source whose output impedance is 86.0 + *j*0 ohms. Recall that when the λ/2 line is terminated with a 100-ohm load resistor the input impedance is reduced to 86.0 ohms due to the 1.0 dB line attenuation. Consequently, the source impedance is matched to the *input* impedance of the line, and the output impedance of the line is matched to its 100-ohm load. The result is that all of the available power from the source is delivered to the load, minus the amount dissipated in the line attenuation. Thus the *output* of the line is now the true source delivering power to the load, and it is delivering to the load all of the power that is available at the line output. Ergo, there is a conjugate match by definition between the source and the line input and between the output impedance of the line and the load impedance (Axioms 1 and 2) *despite the 1.0-dB attenuation* in the line.

Sec APA.6 Disastrous Results of Using Wrong Polarity of Attenuation Factor

Keep in mind from statements in the paragraph above that the network-output impedance is that which allows all of the power available at the network output to be delivered to the load. So let's now see what happens when we **fail to reverse the polarity** of the attenuation factor from positive to negative when transforming the input impedance to the output. The error resulting from this common failure is that which leads to the erroneous result when attempting to measure the output impedance looking rearward from output terminals. To measure the impedance looking rearward from the output terminals the active, non-dissipative source resistance of 86.0 ohms must be replaced with a *dissipative* physical resistor of 86.0 ohms at the line input, and the source for the measurement is applied to the line output. **The result is that both input and output of the line have been interchanged.** From the previous discussion we know there is a conjugate match between the output and the load of 100 + *j*0 ohms, but will the impedance measured rearward at the output be 100 ohms? Because we have now interchanged the input and output terminals of the line to accommodate the measurement, the *direction of travel is reversed*. The 86.0-ohm resistor placed at what was the input is now the *load*, because in making the rearward impedance measurement we are now supplying source power at what was the output, but what is now the *input* during the measurement. Keep in mind that when going from load to input the attenuation factor is *always positive,* thus reducing the SWR and impedance. But during rearward measurement we are again measuring at an **input**, because the original input and output are interchanged. Thus, we are again using *positive* attenuation in going from what is now the load to what is now the input, when we have already used positive attenuation in determining the 86.0-ohm resistance at the original line input with 100 ohms at the load. Consequently, we have *added another* 1.0 dB to the initial attenuation instead of subtracting 1.0 dB in returning to the output as required when returning to the output for a net attenuation of zero. (This is known as 'double dipping' the attenuation.) So let's see what the output measurement will show using the *additional* 1.0 dB attenuation instead of correctly **subtracting** the 1.0 dB. With the 86.0-ohm resistor now terminating the original input as the load, both the resistance and SWR are lower yet at the new input terminals. The resistance now measured is 76.6 ohms and the SWR is 1.533:1, instead of 100 ohms and 2:1 SWR. Not quite what we expected, is it, because we expected the output resistance to be 100 ohms to verify the conjugate match at the load. But there really is a conjugate match, and this resistance of 76.6 ohms must be correctly interpreted to avoid believing incorrectly that the 76.6 + *j*0 ohms proves there is no conjugate match to the 100-ohm load. What has occurred is that the initial 1.0-dB attenuation reduced the 100-ohm load at the output to 86.0 ohms at the input, and the additional 1.0 dB (positive) has reduced the output resistance still further, from 86.0 ohms to 76.6 ohms. If the 1.0 dB had been used correctly as *negative* on the return to the load instead of positive, the output resistance would have returned to 100 ohms and the SWR would have returned to 2:1. Unfortunately the 1.0 dB attenuation factor cannot be negative using this procedure for *rearward measurement,* because, as described earlier, the input and output ports have been interchanged *inherently* during the rearward measurement. Thus the direction of wave travel is reversed with respect to normal direction of operation, *forcing* the attenuation factor to be *positive* during the rearward measurement, resulting in the erroneous result.

Sec APA.7 Emphasis on Difference between Rearward Measurement and Calculation

However, when *calculating* the output impedance from the input impedance we are not constrained to interchange the input and output ports as we are during rearward *measurement*. In calculating the output impedance from the known input impedance we can, **and must** use *negative* attenuation in the direction returning to the load, which yields the correct 100 + *j*0 ohms as the output impedance. On the other hand, if we were to calculate the output impedance with the 86.0-ohm resistance at the input, and then fail to reverse the attenuation factor from positive to negative, we also arrive at the same incorrect 76.6-ohm output resistance obtained with rearward physical measurement. This failure to use negative attenuation factor during the calculation is prevalent, resulting in widespread error in attempting to determine the output impedance of lines and networks. Thus it must be understood that for the rearward *measurement* we **are** constrained to using *positive* attenuation *twice*, once in the initial travel from the load (100 ohms) to the input (86.0 ohms), and again in returning to the load, for a total attenuation of 2.0 dB. (Again, double dipping.) This is because placing the physical 86.0-ohm resistor at the input forced the input to become the terminating load, thus reversing the direction of travel from the desired direction toward the output. It is this doubling of the attenuation factor, instead of compensating it to zero during the return to the load that causes the resulting difference between the measured output impedance and the correct output impedance that misleads one to believe there is no conjugate match when attenuation is present. This is precisely the error Steve Best made in his calculations,[140, 141] from which he claims the principles of impedance matching as discussed in Reflections 1 are erroneous and misleading.

Sec APA.8 Verification of Concept by Application of the Smith Chart

Let's now perform the same calculation on a Smith chart shown in Fig Appendix

9.1 to verify the procedure and calculations performed above. By standard convention, traveling from the load toward the source on the Smith chart uses *clockwise* rotation and *positive* attenuation factor. Positive attenuation causes the impedance curve to spiral inward logarithmically while going from load to source, indicating a shorter chart radius at the source, corresponding to the lower magnitude of the reflection coefficient. Conversely, traveling from the source toward the load uses *counterclockwise* rotation and *negative* attenuation factor, causing the plot to spiral outward on returning to the starting point, the load.

In calculating the input impedance of the λ/2 (180°) transmission line having 1.0 dB of attenuation we begin at the point on the chart that represents the 100 + *j*0 ohm load, 2 + *j*0 chart ohms when normalized to 50 ohms. This point is shown in Fig Appendix 9A.1 at the 2.0:1 point on the resistance axis of the chart where the 100-ohm load resistance appears. The chart radius at this point is 0.33333, equal to the magnitude of the voltage reflection coefficient of the 2:1 mismatch resulting from the 100-ohm load. We now move 360° **clockwise** on the chart**,** representing 180° of transmission line, (positive attenuation) along the **solid** spiral curve on the chart, returning to the resistance axis at the point of 1.72 mismatch that represents the input impedance of 86.0 + *j*0 ohms. The 1.0-dB line attenuation has caused the radius of the spiral curve to be reduced logarithmically to 0.26478 on reaching the 86-ohm point, the amount equal to the 1.0-dB attenuation factor, 0.79433 times the 0.33333 distance to the point with zero attenuation. Remember that the radius at any point on the chart is equal to the voltage reflection coefficient at that point. With zero attenuation the point of arrival at the load would have indicated 100 + *j*0 ohms with the original radius, 0.33333.

However, to determine the output impedance we must first determine the load impedance from our newly found input impedance. The output impedance is the conjugate of the load impedance. We begin at the 86.0-ohm input point just found and retrace the original solid spiral path, reversing the direction of rotation to travel 360° **counter**clockwise (*negative* attenuation) toward the load. The return travel thus spirals outwardly to return to the original starting point (the load) on the chart, 100 + *j*0 ohms, because the output impedance of the line has remained constant. Consequently, to return to the original starting point on the chart we must use **negative** attenuation to compensate for the positive attenuation used in the clockwise direction to determine the input impedance. Ergo, as previously stated, the net attenuation for the round trip—load-to-input and then input-to-load—is **zero**.

However, had we used positive attenuation for the return trip **the original travel would have continued for another 360° in clockwise rotation**, continuing from where the original spiral ended at the 86.0-ohm point. The continuation of the original clockwise spiral is the path shown by the **dashed** curve in Fig Appendix 9A.1 for a second 360° of travel, again returning to the resistance axis at the point of 1.533 mismatch. However, due to the continuation of the spiral inward, the radius has been reduced still further to 0.21023, where the impedance is 76.6 + *j*0 ohms. The radius at this point is 0.74933 (decimal equivalent of 1.0 dB) times shorter than the length at 86.0 ohms, and 0.63096 (decimal equivalent of 2.0 dB) times shorter than the original radius, instead of returning to the original radius of 0.33333 at the 100-ohm point. The point at this shortest radius of 0.21023, represents the impedance 76.6 + *j*0 ohms

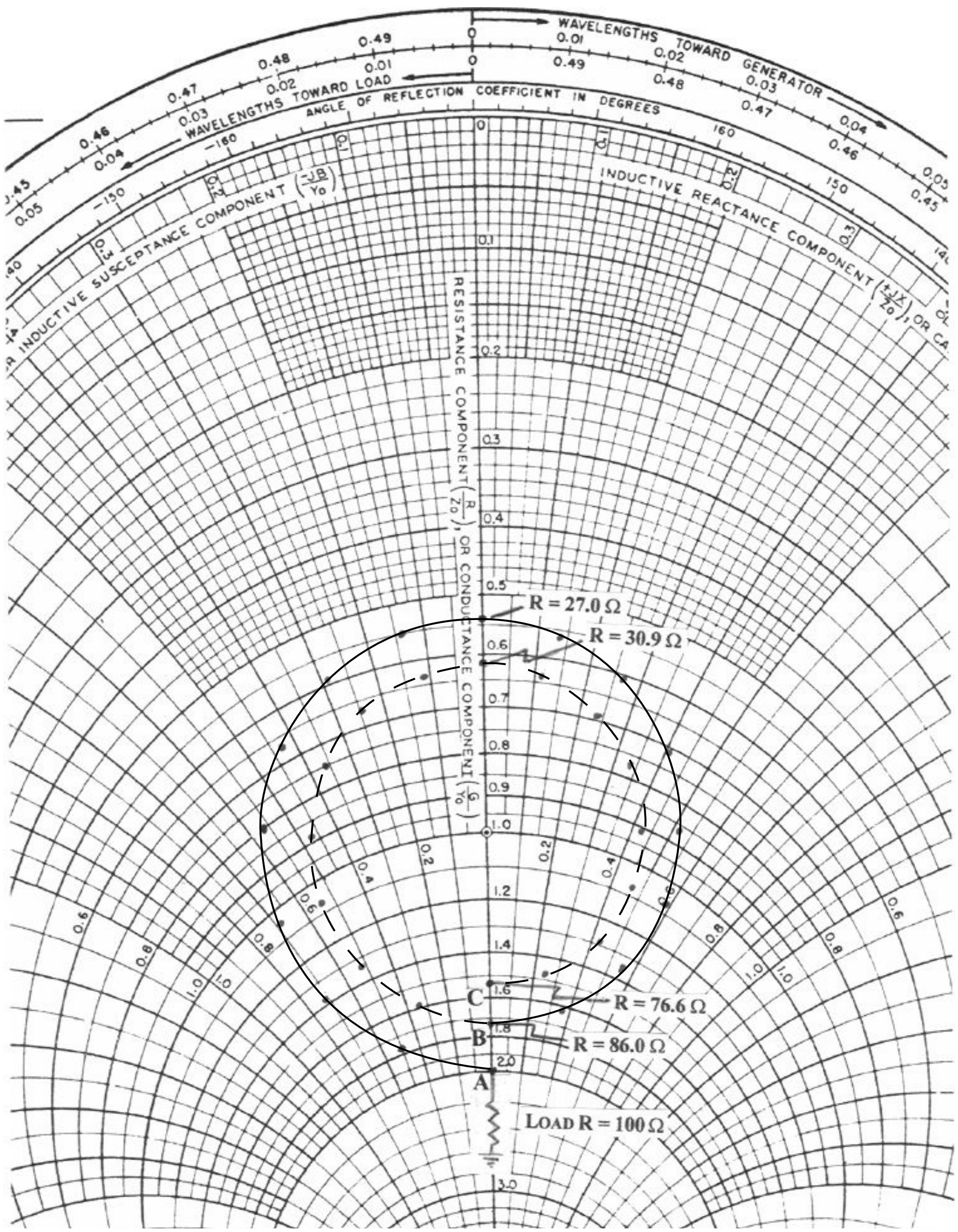

Fig Appendix 9A.1 — Plot of impedances along a 50-ohm, half-wavelength (180º) transmission line, line attenuation 1.0 dB, terminated in pure resistance R = 100 + *j*0 ohms. Clockwise travel along spiral curve from A to B determines line-input resistance 86.0 ohms from load resistance 100 ohms; counterclockwise return travel from B to A determines load resistance from line-input resistance. Placing resistance 86.0 ohms at input of transmission line for rearward measurement to determine output impedance actually places the 86.0-ohm resistor as a new load resistance at B. Consequently, clockwise travel from B to C yields new line-input resistance 76.6 ohms, not line-output impedance as is prevalently considered. This incorrect procedure for determining output impedance of lines inherently requires clockwise travel along the spiral curve, resulting in an erroneous and meaningless value, prevalently considered to be the output impedance. It is imperative that counterclockwise travel be used from B to A to determine load impedance from input impedance — then output impedance is the conjugate of the load impedance, as explained in the text.

as determined in the earlier example where positive attenuation was used twice.

Sec APA.9 Practical Effect of Difference between Correct Output Impedance and Incorrect Impedance Obtained by Rearward Measurement

Before concluding this topic, here is an interesting exercise to determine the effect of the difference (error) between using the measured impedance and the correct output impedance in calculating the transfer of power to the load. We first determine the ratio of the correct output impedance and the measured impedance, and consider this ratio as a *virtual* mismatch. We then determine what the difference in power delivered to the 100-ohm load would be if the measured impedance were the correct output impedance. In other words, what would be the effect if the measured output impedance of 76.6 ohms were considered to be mismatched to the 100-ohm load? Using the data from the example above, the measured impedance resulting from using the 1.0 dB attenuation twice, or 2.0 dB, yields a virtual mismatch of 100 ÷ 76.6 = 1.305:1. This equates to a voltage reflection coefficient magnitude ρ of 0.13250, a power reflection coefficient ρ^2 of 0.01756, and a power transmission coefficient $(1 - \rho^2)$ of 0.9824. This indicates a 98.24 percent delivery of power to the load, representing a difference of 0.077 dB from that which is actually delivered to the load. Thus this infinitesimal difference in power that would be delivered between using the measured impedance and the correct output impedance is insignificant. But it must be remembered that this difference (error) *increases with attenuation*. Consequently, when one uses the measured impedance in attempting to disprove a conjugate match between output impedance and load, don't be misled to believe a conjugate match doesn't exist simply because the measured impedance isn't the exact mathematical conjugate of the load impedance.

Sec APA.10 Conclusion

Finally, contrary to the prevalent misconceptions recited at the beginning, the preceding discussion and examples have shown that attenuation in a network or transmission line does not prohibit a conjugate match from existing in any RF system containing such a network or transmission line.

The following are axioms to remember concerning measurement of the output impedance of networks and lines:

1) Rearward measurement from the output of a line or a network with attenuation does not yield the correct output impedance.
2) Calculation of output impedance from input impedance using positive attenuation factor does not yield the correct output impedance.

To summarize the procedure described above it is appropriate to set forth some rules for guidance when transforming impedances from one end to the other on networks and transmission lines. It will be helpful in understanding the basis for these rules to remember that in transforming impedances from load to input, both the magnitude of the reflected waves and the SWR decrease, due to network or line attenuation, and when transforming from input to load the magnitude and SWR increase. Now the rules:

1) When transforming impedance in the direction from load to input the attenuation is **always** positive.

2) When transforming impedance in the direction from input to load the attenuation is **always** negative.

3) When transforming load impedance to input impedance both phase and attenuation are positive.

4) When transforming input impedance to load impedance both phase and attenuation are negative.

5) When transforming input impedance to output impedance phase is positive and attenuation is negative.

Further discussion on this topic appears in Chapter 24, and computer and hand-held calculator programs for calculating impedance transformations in both directions on transmission lines appear in Chapter 15.